Perspectival Disagreement

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Overview

Preference Perspectivity

The Meal Example

Suppose John and Mary have the following preferences over a set of main dishes:

	John		Mary
1	salad	204	steak
2	goulash	203	pizza
⋮	⋮	⋮	⋮
203	pizza	2	goulash
204	steak	1	salad

John and Mary disagreement is *perspectival* if the choice between salad and goulash is more important for John than for Mary provided that only these dishes and perhaps a few other dishes lower in John's ordering are available (and correspondingly for Mary, steak and pizza).

Does It Matter?

The Terror Response Example

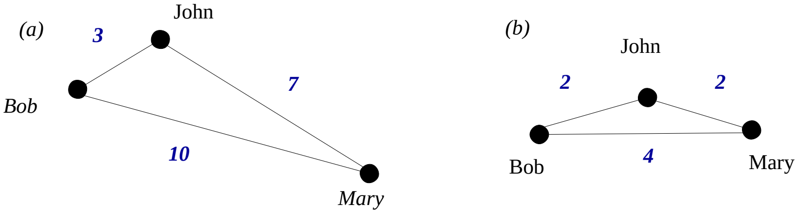
Suppose first there are three politicians who disagree about the priorities in response to a major terrorist attack. The options are: 1 - install more CCTV cameras, 2 - increase the budget and training of rapid response police forces, 3 - destroy the financial sources of the terrorist group in question, or 4 - raise the budget of intelligence agencies.

Agent	Ordering	Labeling
John	$b \succ d \succ a \succ c$	1 2 3 4
Mary	$d \succ b \succ a \succ c$	2 1 3 4
Bob	$b \succ d \succ c \succ a$	1 2 4 3

Is John nearer to Bob than to Mary?

Note: Obviously these options are non-exclusive but suppose, for the sake of the argument, that they represent exclusive priorities.

Illustration



Disagreement between John, Bob and Mary according to (a) a modified Footrule measure, and (b) ordinary Footrule. In this case, application of the modified measure is still symmetric. For example, the distance of John to Mary equals the distance of Mary to John. However, this is not the case in general.

Permutation Notation

As a way to keep definitions simpler, the strict preferences of one agent can be written as a permutation of those of another agent:

Mapping	Permutation	Preferences
John $\mapsto$ Mary	$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$	$a \succ b \succ c \succ d$ $b \succ c \succ d \succ a$

Pairwise Distance Measures / Consensus Measures

1. Spearman's Footrule sums the absolute distance of the ranks (Spearman 1904):

$$F(\pi) = \sum_{i}^{n} |i - \pi_i|$$

2. Kendall's Tau counts twice the number of inversions (adjacent transpositions) needed to get from one preference to the other (Kendall 1938):

$$K(\pi) = \sum_{i}^{n-1} |\{\langle i, j\rangle \mid j > i \;\; \& \;\; \pi_i > \pi_j\}|$$

3. Bogart (1973) generalizes Kendall's Tau to incomplete orders.
4. See also Critchlow (1986) on incomplete orders and preorders.

Positional Variants

For the Footrule measure:

$$F^*(\pi) = \sum_{i}^{n} |i - \pi_i|(n + 1 - i).$$

In general, from

$$D(p, q) = \sum_{i=1}^{n} \Delta(i, p, q)$$

we obtain

$$D^*(p, q) = \sum_{i=1}^{n} \Delta(i, p, q)(n + 1 - i),$$

where Δ is a linear local distance function for each level of the ordering and p, q are the preferences.

Positional Variants Are Not Distance Measures

Conditions for a Distance Measure (Deza&Deza 2009: 16)

$D(x, y) = 0 \Leftrightarrow x = y$	Coincidence
$D(x, y) = D(y, x)$	Symmetry
$D(x, z) \leq D(x, y) + D(y, z)$	Triangle Inequality

It is easy to find examples of applications of the positional variants that violate symmetry.

Counter-Example

Lack of Symmetry

Mapping	Permutation	Preferences
John $\mapsto$ Mary	$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$	$a \succ b \succ c \succ d$ $b \succ c \succ d \succ a$
Mary $\mapsto$ John	$\pi^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix}$	$b \succ c \succ d \succ a$ $a \succ b \succ c \succ d$

$$F(\pi) = F(\pi^{-1}) = 6$$

$$F^*(\pi) = 12$$

$$F^*(\pi^{-1}) = 18$$

What Does That Mean?

Should symmetry fail?

- If we express Mary's ordering as a permutation of John's ordering, and calculate a positional measure, this expresses John's distance to Mary based on the relative importance derived from *his* values.
- If we express John's ordering as a permutation of Mary's ordering, and calculate a positional measure, this expresses Mary's distance to John based on the relative importance derived from *her* values.

If our goal is to capture this perspectivity, then the measure should *not* be symmetric w.r.t. the inverse permutation, hence not be a proper distance measure.

Observer-dependent Disagreement

Introduce an observer who 'filters' observed value orderings through his value orderings:

$$D_r(p,q) = \sum_{i=1}^{n} \Delta(i, \pi(r,p), \pi(r,q))(n+1-i), \tag{1}$$

where $\pi(r,p)$ is the permutation that expresses the preferences p as a permutation of the strict preferences r. For instance, with some abuse of notation, an observer-relative version of Spearman's footrule may be defined by

$$F_r(p,q) = \sum_{i=1}^{n} |\pi[r,p](i) - \pi[p,\pi(r,q)](i)|(n+1-i). \tag{2}$$

In this setting, two observers may disagree about the distance between two or more agents, even though they base their judgment on the same evidence.

Generalized Positional Disagreement

How about making the score at each level arbitrary?

- These measures have been investigated for search engine rankings (Kumar & Vassilvitskii 2010).
- But the definitions in this literature are defined in a way that keeps symmetry, so they remain distance measures.
- Perhaps not for every application the functions should be distance measure.

What about cardinal utility representations of preferences?

- If the representation is cardinal, utility differences are meaningful.
- These differences may give rise to a generalized score at each level.
- If so, cardinal utilities give rise to perspectival disagreement.

Conclusions

- Perspectival disagreement might sometimes occur.
- For cases when it occurs using positional measures of disagreement seems to be adequate.
- The modified functions are a natural generalization of known distance measures, though not proper distance measures themselves.
- Non-symmetry of the modified versions is intuitively supported from the way we can understand value perspectivity.
- However, the relative importance of an inversion ('preference swap') might not always depend on the position alone.

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Appendix A: Another Example

Example 2

Suppose that there is compelling evidence that a nation is violating a denuclearization treaty and is in the process of building nuclear weapons. Let the alternative be: 1 - impose unilateral sanctions, 2 - form a coalition and increase international diplomatic pressure, 3 - threaten the country with a military intervention, and 4 - conduct air strikes against weapon manufacturing plants without prior warning.

Agent	Ordering	Labeling
John	$b \succ d \succ a \succ c$	1 2 3 4
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Because of the high stakes of the lower options, importance may be decoupled from the value ordering – or perhaps two separate value dimensions are aggregated.

Appendix B: Maximum Values

Let n be the the cardinality of the union of the domains of the respective preorders for Bogart's measure and its positional variant, the size of the linearized preorders for the other measures. The maximum values in terms of n are:

- Footrule Distance (Diaconis & Graham 1977: 264): $\lfloor n^2/2 \rfloor$.
- Kendall's Tau (Kendall 1970), Bogart Distance (Bogart 1973: 64): $n(n-1)$.
- Positional Footrule: $\frac{1}{2}a(-1+a-n)(a-n)+\frac{1}{6}(-a-3a^2+4a^3+3an-9a^2n+6an^2)$, where $a = \text{round}(3n/7)$.
- Positional Bogart: $2kn(k+2)/3$, where $k = max(n-1, 0)$.